Math 286
Differential Equations

Section 1.5 Handout
Kate Bella

Existence & Uniqueness Theorem. Suppose $f(t, y)$ is a continuous function on the rectangle $R = \{(t, y) : t \in (a, b), y \in (c, d)\}$. Then for each point (t_0, y_0) in R there exists an $\epsilon > 0$ and a function $y(t)$ defined on $t \in (t_0 - \epsilon, t_0 + \epsilon)$ that solves the IVP

$$\begin{cases} \dfrac{dy}{dt} = f(t, y) \\ y(t_0) = y_0 \end{cases}$$

If additionally $\dfrac{\partial f}{\partial y}$ is continuous in R and $y_1(t)$ and $y_2(t)$ are two functions that solve the IVP in some interval $(t_0 - \epsilon, t_0 + \epsilon)$ for some positive number ϵ, then $y_1(t) = y_2(t)$ in that interval.